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Cyclotron resonance of electrons in a semiconductor quantum well

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Abstract. The cyclotron resonance of electrons confined in a quantum well was investigated by means of the Wigner–Brillouin perturbation theory. It is found that the splitting of Landau levels can occur near the frequency of the long-wavelength transverse-optical phonons of the well barriers owing to the interface phonons. Decreasing the width of the well increases the electron–phonon interaction. Electronic screening weakens the interaction.

1. Introduction

Recently there has been considerable interest in the effects of phonon confinement on the electron–phonon interaction. In the case of a bulk semiconductor, the cyclotron resonance of electrons in a magnetic field has been extensively studied experimentally and theoretically. The Landau level splitting takes place near the frequency of long-wavelength longitudinal-optical (LO) phonons. The situation in confined geometries is considerably less clear. The cyclotron resonance experiments have shown that the 'pinning' occurs near the frequency of the long-wavelength transverse-optical (TO) phonons in heterojunctions [1, 2] and in quantum wells [3, 4].

It is worthwhile emphasizing that, in Ziemann's [4] quantum well (InAs/GaSb) experiment, the 'pinning' takes place near the frequency of the TO phonons in the well barriers (GaSb) where just the tail of the electronic wavefunction exists.

These results have been interpreted in terms of the superlattice folding of the LO phonons [5], non-parabolicity of the cyclotron mass [6, 7] and resonant polaron effects [2, 8]. However, perplexing questions still remain.

In previous work [9], one of the present authors and a co-worker investigated the optical phonon modes in a quantum well and the interaction of an electron with these phonons. The electron–phonon interaction Hamiltonian has been given. There are five branches of LO phonons in InAs/GaSb quantum wells. One of them is the bulk mode with the LO-phonon frequency of the well layer. It is located within the well region. The other four branches are interface phonons. Two of them vibrate with the frequencies of the LO and TO phonons of the well layer. We call them the LO_1- and TO_1-interface phonons. The other two branches occupy the frequencies of LO and TO phonons, respectively, of the well barriers. We call them the LO_2- and TO_2-interface phonons. It can be understood that the pinning effect takes

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place near the frequency of TO phonons of the well barriers because TO_2 phonons exist in the quantum wells.

In this paper, we discuss the interaction of an electron with the TO_2-interface phonons in a quantum well under a magnetic field and take the screening effect of the two-dimensional electron gas (2DEG) into account. In section 3, the expression for the energy of the $n = 1$ Landau level will be given by means of the Wigner–Brillouin perturbation theory. The cyclotron resonance frequency of the InAs/GaSb quantum well will be calculated numerically and some conclusions will be discussed.

2. Hamiltonian

We consider a thin quantum well consisting of a polar semiconductor 1 sandwiched between two thick layers consisting of a polar semiconductor 2. The width of the well is $2d$. The magnetic field is perpendicular to the well plane. The Hamiltonian of the system of an electron and the phonons can be written as

$$H = H_0 + H_I \tag{1}$$

$$H_0 = \frac{1}{2m}\left(p_x - \frac{eB}{m}\right)^2 + \frac{1}{2m}p_y^2 + \frac{1}{2m}p_z^2 + U(z) + \sum_k \hbar\omega(a_k + a_k^+)$$

$$U(z) = \begin{cases} 0 & \text{if } |z| < d \\ \infty & \text{otherwise} \end{cases}.$$

Here m is the band mass of the electron, B is the magnetic field strength, and a_k^+ and a_k are the creation and annihilation operators, respectively, of the phonons. We focus on the interaction of the electron with the TO_2-interface phonons and investigate the behaviour of the electron when the cyclotron resonance frequency ω_c is near the frequency ω of the TO_2-interface phonons. Thus, according to [9], the interaction Hamiltonian is

$$H_I = \sum_k F(k, z)\exp(ik \cdot \rho)(a_k + a_{-k}^+) \tag{3}$$

where

$$F(k, z) = i\left(\frac{2\pi\hbar e^2}{S\omega}\right)^{1/2} \frac{\epsilon_2\Delta_1 - \epsilon_1\Delta_2}{(\epsilon_1 - 1)(\epsilon_2 - \epsilon_1)} \frac{2A_-(k)}{k} \sinh(kz) \tag{4}$$

and

$$\Delta_j = \frac{\omega_{Tj}(\epsilon_{0j} - \epsilon_{\infty j})^{1/2}(\epsilon_j - 1)^2}{\epsilon_j - \epsilon_{\infty j}} \qquad j = 1, 2$$

$$\omega^2 = \frac{B(k) - [B(k)^2 - 4A(k)C(k)]^{1/2}}{2A(k)}$$

$$A(k) = (\epsilon_{\infty 1} + \epsilon_{\infty 2})(\epsilon_{\infty 1} - \epsilon_{\infty 2})\exp(-2kd)$$

$$B(k) = \epsilon_{\infty 1}(\omega_{L1}^2 + \omega_{T2}^2)[1 + \exp(-2kd)] + \epsilon_{\infty 2}(\omega_{L2}^2 + \omega_{T1}^2)[1 - \exp(-2kd)]$$

$$C(k) = \epsilon_{\infty 1}\omega_{L1}^2\omega_{T2}^2[1 + \exp(-2kd)] + \epsilon_{\infty 2}\omega_{L2}^2\omega_{T1}^2[1 - \exp(-2kd)]$$

$$A_-(k) = \left(\frac{k/2}{\{[(\epsilon_1 + 1)/(\epsilon_1 - 1)]\exp(2kd) + 1\}^2\exp(-2kd) + \sinh(2kd)}\right)^{1/2}.$$

In the above expressions, ϵ_0 and ϵ_∞ are the static and high frequency dielectric constants, respectively, ω_L and ω_T are the LO- and TO-phonon frequencies, respectively, and ϵ_j are the frequency-dependent dielectric constants of the media 1 and 2, given by

$$\epsilon_j = \frac{\omega_{LO}^2 - \omega^2}{\omega_{TO}^2 - \omega^2} \qquad j = 1, 2.$$

In [9], the frequency of the TO_2-interface phonons and the related interaction function are labelled $\omega_{--}(k)$ and $F_{--}(k, z)$. Here we ignore the indexes for convenience. $F(k, z)$ is the electron–phonon coupling function which is determined within the wells. Its absolute value becomes a maximum at $z = \pm d$ and a minimum at the centre of the well. They decrease exponentially far away from the interfaces outside the wells [9]. However, the electron discussed here does not interact with the phonons outside the well because its wavefunction is located within the well.

Electron–electron interaction will screen the electron–phonon interaction. This screening considerably weakens the electron–phonon interaction [10]. Taking the screening into account, the electron–phonon interaction Hamiltonian becomes

$$H_I = \sum \frac{F(k, z)}{\epsilon(k, \omega)} \exp(ik \cdot \rho)(a_k + a_{-k}^+) \tag{5}$$

where $\epsilon(k, \omega)$ is the dielectric function of the 2DEG. Next, the dielectric function will be approximated by its static value $\epsilon(k, 0)$. This approximation is allowed in treating the screening in heterostructures and quantum wells [11, 12]. Thus, the electron–phonon interaction can be written

$$H_I = \sum \frac{F(k, z)}{\epsilon(k, 0)} \exp(ik \cdot \rho)(a_k + a_{-k}^+). \tag{6}$$

Here $\epsilon(k, 0)$ is the static dielectric function of the 2DEG. It has a closed form in the random-phase approximation (RPA) [11]:

$$\epsilon(k, 0) = 1 + \frac{4me^2}{\hbar^2 \epsilon_\infty} \frac{\nu}{k} F_0\left(\frac{\hbar k^2}{2m\omega_c}\right) \qquad \text{for } \nu < 1 \tag{7}$$

$$F_0(a) = \exp(-a) \int_0^a dx \, \frac{\exp x - 1}{x}$$

$$\epsilon(k, 0) = 1 + \frac{4me^2}{\hbar^2 \epsilon_\infty} \frac{1}{k} \left[F_0\left(\frac{\hbar k^2}{2m\omega_c}\right) + (\nu - 1) F_1\left(\frac{\hbar k^2}{2m\omega_c}\right) \right.$$

$$\left. - (\nu - 1) \frac{\hbar k^2}{2m\omega_c} \exp(-\hbar k^2 / 2m\omega_c) \right] \qquad \text{for } 1 \leqslant \nu \leqslant 2 \tag{8}$$

$$F_1(a) = [1 - \exp(-a)] + (1 - 2a) F_0(a) + a^2 \exp(-a) \int_0^a dx \, \frac{\exp x - 1 - x}{x^2}.$$

Here $\nu = (n_e/2)(2\pi\hbar/m\omega_c)$ is the filling factor and $\omega_c = eB/mc$ is the cyclotron resonance frequency when no polaron effects are present.

3. Calculation and conclusion

In equation (1), H_0 is the unperturbed Hamiltonian, H_I is the perturbation. For simplicity and without loss of generality, we take $P_x = 0$ and assume that the electron occupies the ground state of the z direction consistently. The unperturbed eigenstates of H_0 are

$$\psi_n = \phi(z)\phi_n(y) \tag{9}$$

$$\phi(z) = \begin{cases} \left(1/\sqrt{d}\right)\cos(\pi z/2d) & \text{if } |z| < d \\ 0 & \text{otherwise} \end{cases}$$

$$\phi_n(y) = \left(\frac{(eB/m)^{1/2}}{\sqrt{2\pi}2^n n!}\right)^{1/2} H_n[(eB/2m)^{1/2}y]\exp(-eB/4my^2)$$

where H_n is the Hermite polynomial. The wavefunction (9) is coupled via H_I to the perturbed wavefunction of the form $\phi_n(z)\exp(-ik_x x)\ \phi_n(y - \hbar k_x/eB)|k\rangle$, where $|k\rangle$ is the phonon state with the wavevector k.

In the Wigner–Brillouin perturbation theory, the energy of the perturbed $n = 1$ state is

$$E_1 = \tfrac{3}{2}\hbar\omega_c - \sum_k \frac{|\langle\phi(z)|F(k,z)|\phi(z)\rangle|^2}{\epsilon(k,0)}\exp(-\hbar k^2/eB)$$
$$\times \sum \frac{1}{k}(\hbar k^2/2eB)^{n-1}\frac{(n-\hbar k^2/2eB)^2}{(n+1/2)\hbar\omega_c + \hbar\omega - E_1}. \tag{10}$$

The most important term in the perturbation sum for $\hbar\omega_c$ near $\hbar\omega$ is the $n = 0$ term, because this term has the smallest energy denominator. Thus near the level cross region a good approximation is

$$E_1 = \tfrac{3}{2}\hbar\omega_c - \sum_k \frac{|\langle\phi(z)|F(k,z)|\phi(z)\rangle|^2}{\epsilon(k,0)^2}\exp(-\hbar k^2/2eB)\frac{\hbar k^2/2eB}{(1/2)\hbar\omega_c + \hbar\omega - E_1}. \tag{11}$$

Using (11) we calculated the Landau levels of the InAs/GaSb quantum well. The results are shown in figures 1 and 2. In figure 1, the differences between the Landau levels $n = 1$ and $n = 0$ ($E_0 = 1/2\hbar\omega_c$) were plotted. The chain dashed curve describes the energy level differences in the well of width $2d = 200$ Å, the chain dotted curve the width $2d = 100$ Å, the dashed curve the width $2d = 40$ Å, the solid curve the width $2d = 20$ Å. The two dotted straight lines in figures 1 and 2 are the one-phonon energy of the TO_2-interface phonon and the Landau level $n = 1$ without the interaction with the phonons. The screening will occur in this energy cross region. It can be seen from figure 1 that the level splitting surely takes place in this region. The level curves of narrower wells are closer to the two dotted straight lines, reflecting more intense interaction between the electron and the phonons.

In figure 2, the curves are the differences between the Landau levels $n = 1$ and $n = 0$ in the same well of width $2d = 100$ Å, but with different densities of 2DEG, namely $\nu = 0.9$ and $\nu = 0$. That the Landau levels with $\nu = 0.9$ are closer to the two dotted lines can be explained by the fact that the electron gas weakened the electron–phonon interaction.

It may be difficult to understand why the pinning effect can take place near the TO-phonon frequency because it may be thought that TO phonons which are transverse fields do not interact with electrons. As mentioned above in section 1, five branches of LO phonons

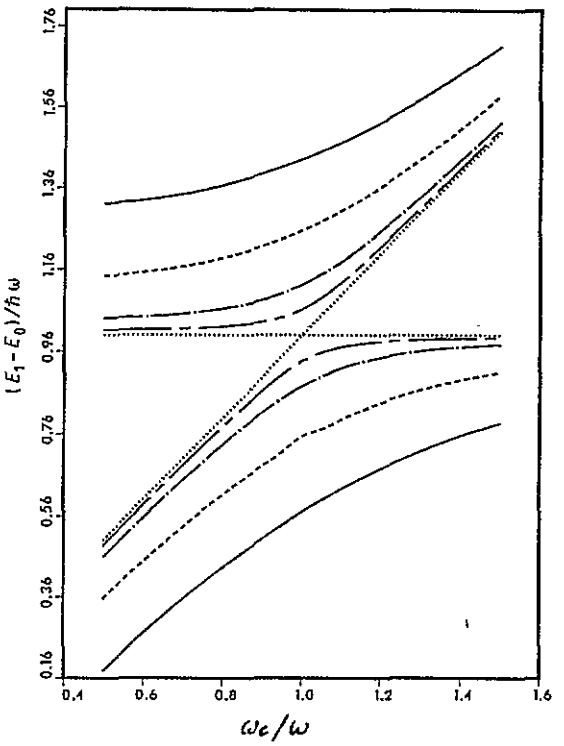

Figure 1. The difference between the Landau levels $n = 1$ and $n = 0$ of a single electron without an electron gas ($\nu = 0$) in quantum wells of different widths. The widths of the wells are 200 Å (— - —), 100 Å (— · —), 40 Å (----) and 20 Å (——). ω_c is the cyclotron resonance frequency and ω is the frequency of the TO_2-interface phonons. In the region centred at $\omega_c = \omega$ the splitting of Landau levels occurs. The energy curves in wider wells are closer to the two crossed straight lines (· · · · · ·), reflecting the weaker interaction between the electron and the phonons.

exist in quantum wells, which can all interact with electrons. Two of them are the interface phonons taking TO_1- and TO_2-phonon frequencies, respectively. The reason why pinning occurs near a TO-phonon frequency lies in the existence of the interface phonons. It is reasonable to mention that Landau level splitting can take place near all these frequencies, namely TO_1-, TO_2-, LO_1- and LO_2-phonon frequencies. So the phonons play an important part in the electron states in quantum wells.

In the above computations for the dielectric function of the electron gas, we take the static RPA results $\epsilon(k, 0)$ neglecting the frequency dependence of the dielectric function. Making this simplification we assume that the phonon frequency is unchanged by the electron gas which is reasonable for $\omega_P \ll \omega_c$, where ω_P is the plasmon frequency of the 2DEG. The condition $\omega_P \ll \omega_c$ is valid for a not too high electron density. In general, the electron density is less than 10^{-12} cm^{-2} in quantum wells and heterojunctions. In the InAs/GaSb quantum well discussed here the energy $\hbar\omega_P$ of the plasmon equals 0.35 meV when the electron gas density $n_e = 10^{-12}$ cm^{-2} and the energy $\hbar\omega$ of the TO_2-interface phonon is 28.59 meV.

In summary, we investigated the cyclotron resonance in quantum wells and found that the splitting of the Landau levels can occur near the frequency of TO phonons of the well

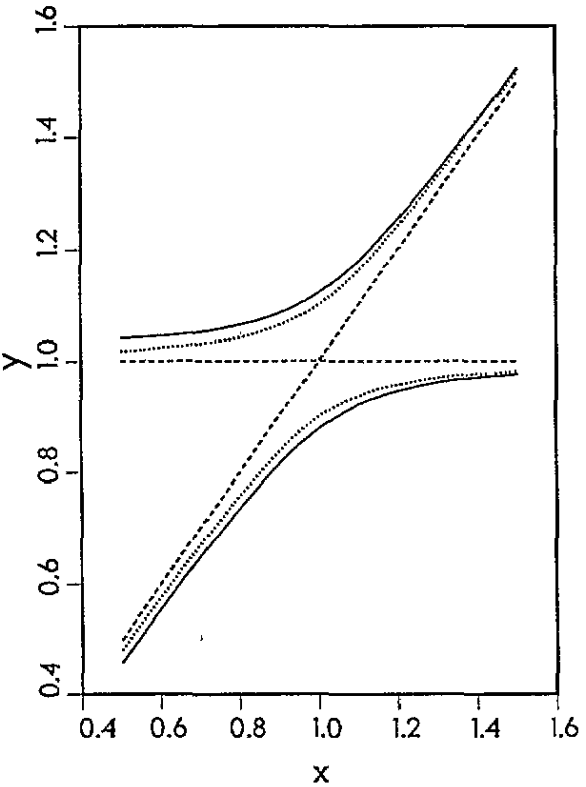

Figure 2. The differences between the Landau levels $n = 1$ and $n = 0$ in the same wide well but for different densities of the electron gas: (- - - -) $\nu = 0.9$; (——) $\nu = 0$. It can be seen that the pinning takes place near the frequency ω and the electron gas weakens the electron–phonon interaction.

barrier owing to the interface phonons. The electron–phonon interaction becomes stronger with decreasing width of the well. The electron gas screening effect weakens the electron–phonon interaction.

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